Gravitation

(Kepler's laws, Newton's inverse square law, g and G)

Note : This PPT will NOT help you learn physics concepts. It is intended only as a quick revision of formulas, definitions, theorems and concepts before examinations. No physics can be learnt just by watching a few videos or going through a few slides of PPT.

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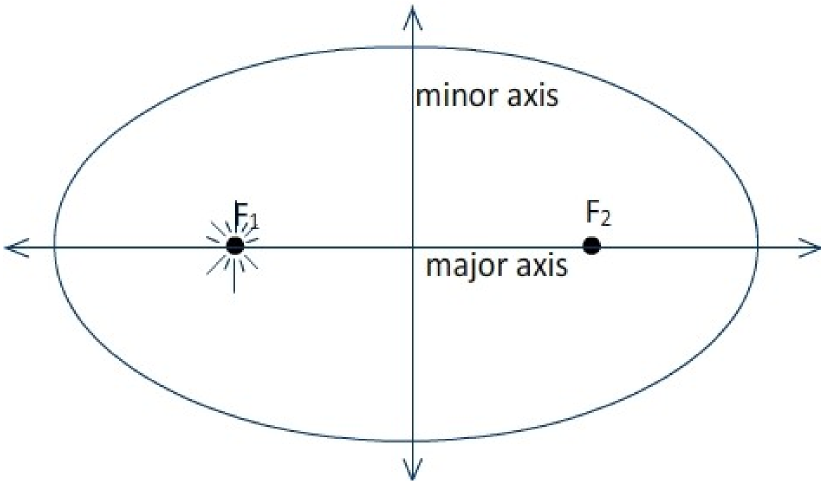
Kepler's laws of planetary motion



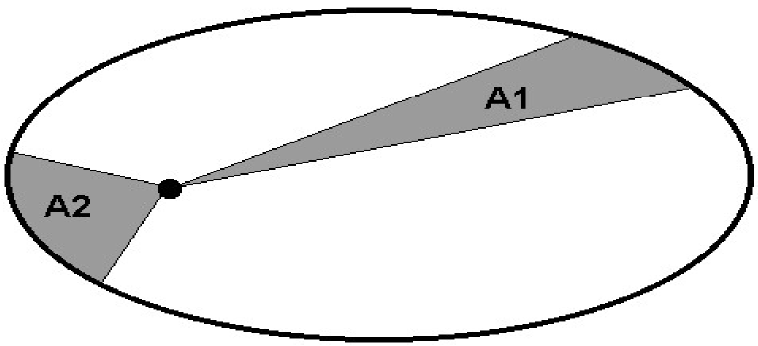
- 1st law (law of orbits)

  Each planet revolves around the sun in elliptical orbits with sun at the one of the principal foci

- 2nd law (law of areas)

  Radius vector of the planet sweeps equal areas in equal intervals of time.

$$\frac{dA}{dt} = \text{constant}$$



- 3rd law (law of periods)

  Square of time period of revolution of the planet is proportional to cube of the mean radius of its orbit.

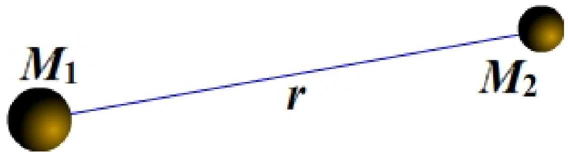
$$T^2 \propto R^3$$

Note : Length of the semi-major axis is taken as the mean radius of the orbit

Newton's inverse square law of gravitation

Gravitational force between two point objects is directly proportional to product of masses, inversely proportional to square of distance between them and acts along the line joining them.

$$\boldsymbol{F} = G\,\frac{M_1 M_2}{r^2}\,\hat{r}$$



G is known as the universal gravitational constant

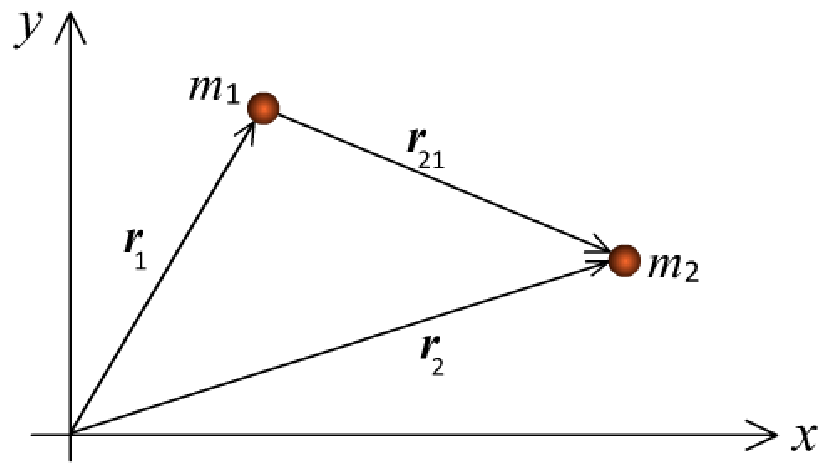
- ❑ Does not depend on the nature of bodies
- ❑ Does not depend on medium between the bodies

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Dimensional formula of G is [$M^{-1} L^3 T^{-2}$]

Vector form of Newton's law of gravitation

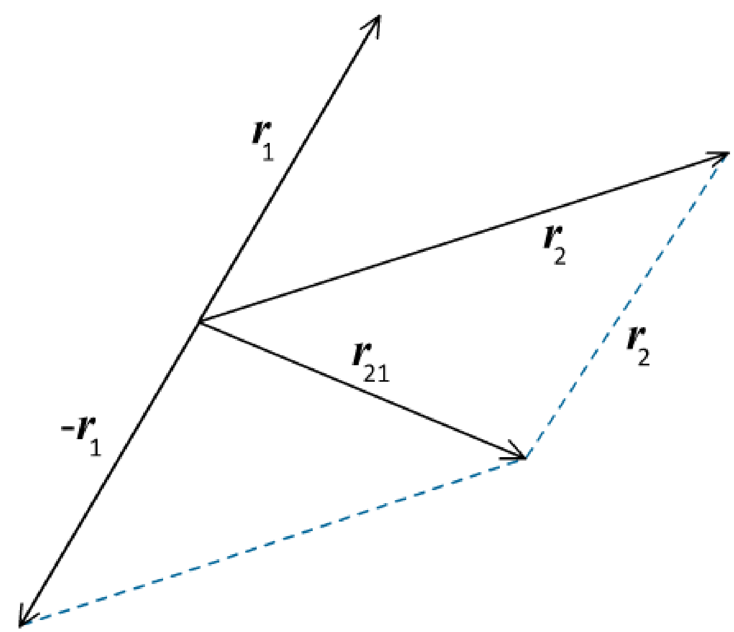
Consider two bodies of masses m_1 and m_2 located at positions r_1 and r_2, respectively, from the origin. The position vector of m_2 w.r.t. m_1 is given by $\boldsymbol{r}_{21} = \boldsymbol{r}_2 - \boldsymbol{r}_1$.



Gravitational force on m_1 (due to m_2) is given in the vector form by

$$\boldsymbol{F} = \frac{Gm_1m_2}{\boldsymbol{r}_{12}^{\;2}}$$

$$\boldsymbol{F} = \frac{Gm_1m_2}{r_{12}^{\;3}}\bar{\boldsymbol{r}}_{12}$$

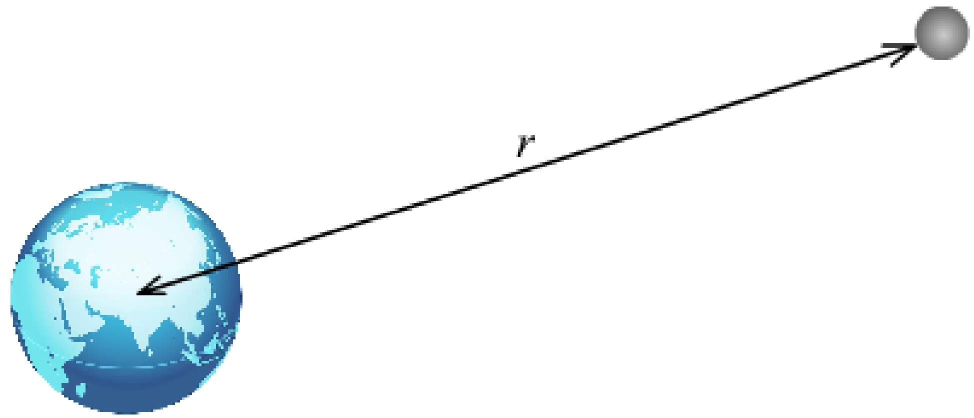


Gravitational force between earth and moon

If the gravitational force of moon on the earth is $\boldsymbol{F}$ then the gravitational force of the earth on moon is $-\boldsymbol{F}$.

$$\boldsymbol{F}_{\text{moon}} = \frac{GMm}{r^2}\hat{\boldsymbol{r}}$$

$$\boldsymbol{F}_{\text{earth}} = \frac{GMm}{r^2}(-\hat{\boldsymbol{r}})$$



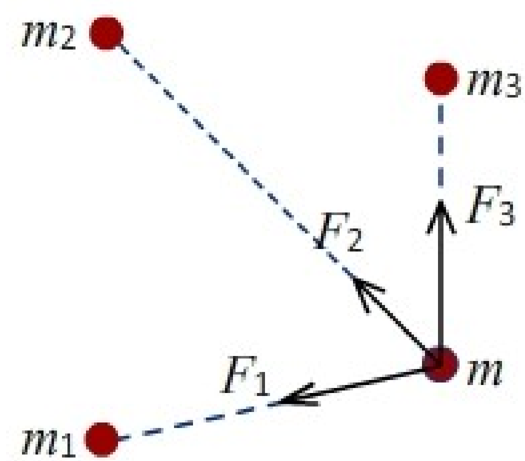
Gravitational forces form action-reaction pair.

Net force due to multiple point objects

The net gravitational force acting on the body is the vector sum of gravitational force on it due to each body near it.

Using principle of superposition

$$\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 \ldots$$

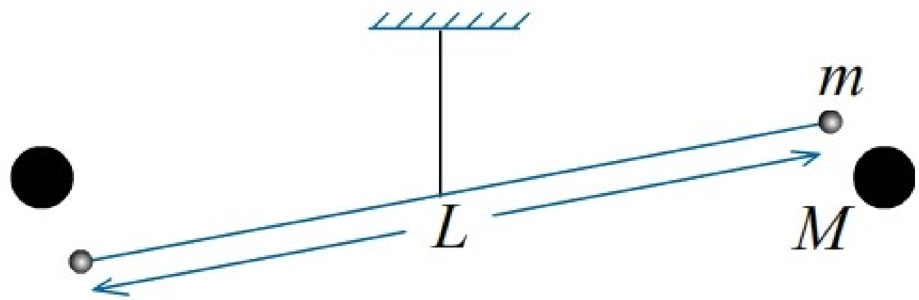


Determination of G (Cavendish experiment)

The experimental setup consisted of a horizontal bar, with small lead spheres attached at each end , suspended by a thin wire. Two large spheres were placed nearby the smaller spheres in opposite directions (as shown in the figure) .

Deflecting torque due to the gravitational force between the large and small spheres twisted the wire.

$$\tau_D = L \times G\frac{Mm}{r^2} \quad \text{—— (i)}$$



As the wire is twisted by an angle θ, the restoring torque (τ_R) due to the wire is

$$\tau_R = C\theta \quad \text{—— (ii)}$$

Where C is the couple per unit twist of the wire which can be determined independently.

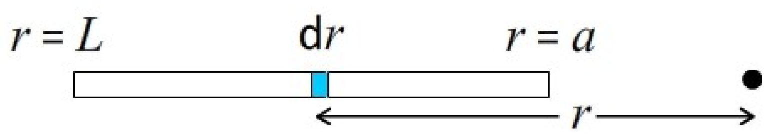
At equilibrium τ_D = τ_R therefore equating (i) and (ii) we get

$$L \times G\frac{Mm}{r^2} = C\theta$$

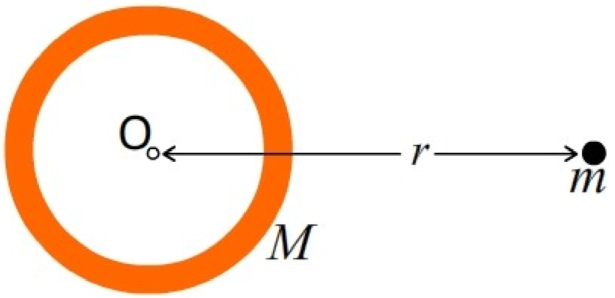
$$G = C\theta\frac{r^2}{L\,Mm}$$

Gravitational forces in case of extended objects

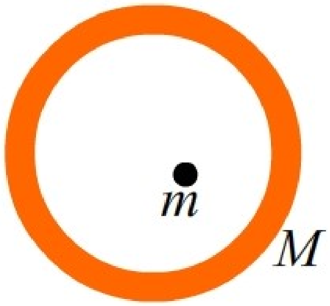
- Newton's law of force between two point masses cannot be used for extended objects.
- For extended objects, force is exerted on (or by) each infinitesimal segment of the object. The total force has to be calculated using integration



- Special case I : The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell.



- Special case II : The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is zero.



Relation between G and g

Consider a body of mass m located very close to the surface of the earth of mass M and radius R.

Gravitational force acting on the body is given by Newton's inverse square law i.e.

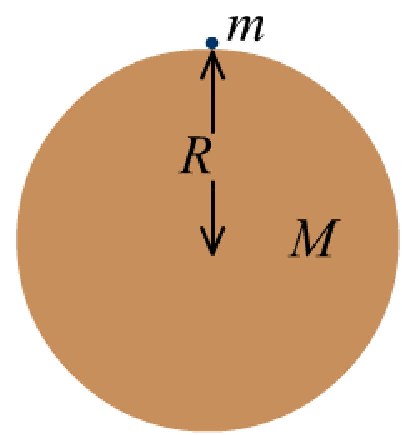
$$F = \frac{Gm_1m_2}{r^2}$$

$$\Rightarrow \quad F = \frac{G\,M\,m}{R^2} \qquad \text{(i)}$$

Using Newton's second law of motion we get

$$F = mg \qquad \text{(ii)}$$

From equations (i) and (ii) we get



$$mg = \frac{G\,M\,m}{R^2}$$

$$g = \frac{GM}{R^2}$$

In terms of density (ρ) of the earth the above relation may be written as

$$g = \frac{4G\rho\pi R}{3}$$

Example

What would be the change in acceleration due to gravity at the surface, if the radius of Earth decreases by 2% keeping the mass of the Earth constant?

Acceleration due to gravity is given by the relation

$$g = \frac{GM}{R^2}$$

As the radius decreases by 2% we get

$$g' = \frac{GM}{\left(\frac{98}{100}R\right)^2}$$

$$g' = \frac{10000}{98 \times 98}\frac{GM}{R^2}$$

$$g' = 1.04 \times \frac{GM}{R^2}$$

$$g' = 1.04\,g$$

Therefore acceleration due to gravity increases by 4 %.

Example

Keeping the length of a simple pendulum constant, will the time period be the same on all planets?

No. Time period will not remain constant.
Time period of oscillation of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Acceleration due to gravity is given by the relation

$$g = \frac{GM}{R^2}$$

As acceleration depends on the mass and radius of the planet, it varies from one planet to another. Therefore time period of oscillation of the pendulum also varies.

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