Gravitation

(variation of g with height , depth and latitude)

Note : This PPT will NOT help you learn physics concepts. It is intended only as a quick revision of formulas, definitions, theorems and concepts before examinations. No physics can be learnt just by watching a few videos or going through a few slides of PPT.

V Vijaykiran
M.Sc., B.Ed., NET
SIGMA Physics Resource Centre
http://www.sigmaprc.in

Variation of g with altitude

Consider a body of mass m located at a height h from the surface of the earth.

Gravitational force acting on the body is given by Newton's inverse square law i.e.

$$F = \frac{GMm}{(R+h)^2} \quad \text{(i)}$$

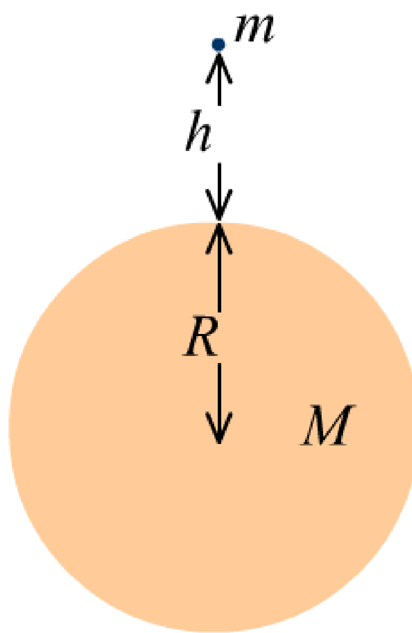
Using Newton's second law of motion we get

$$F = mg_h \quad \text{(ii)}$$

From equations (i) and (ii) we get

$$mg_h = \frac{GMm}{(R+h)^2}$$

$$g_h = \frac{GM}{(R+h)^2}$$



$$g_h = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2}$$

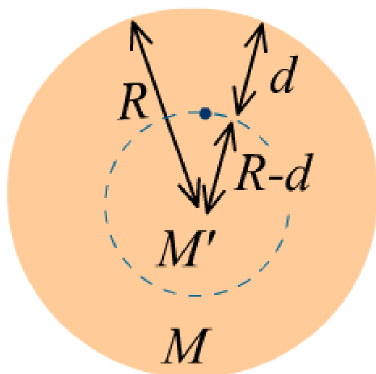
$$g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

When $h << R$ the above relation may be approximated as

$$g_h = g\left(1-\frac{2h}{R}\right)$$

Variation of g with depth

Consider a body of mass m located at a depth d from the surface of the earth.



Using Newton's inverse square law

$$F = \frac{GM'm}{(R-d)^2} \quad \text{(i)}$$

Using Newton's second law of motion we get

$$F = mg_d \quad \text{(ii)}$$

From equations (i) and (ii) we get

$$g_d = \frac{GM'}{(R-d)^2}$$

Considering mass of the earth enclosed only within the sphere of radius ($R-d$) we get

$$g_d = \frac{GM(R-d)^3}{R^3(R-d)^2}$$

$$g_d = \frac{GM}{R^2}\left(1-\frac{d}{R}\right)$$

$$g_d = g\left(1-\frac{d}{R}\right)$$

Acceleration due to gravity, at the centre of the earth, is zero.

Comparison of g at equal depth & height

Consider two points, one at a height (h) and the other at depth (d) from the surface of the earth. Let h be equal to d and denoted as x. Using the condition that h & d << R we get

$$g_h = g\left[1 - \frac{2x}{R}\right]$$

$$\Rightarrow \quad g_h = g\left[\frac{R - 2x}{R}\right] \qquad \text{—— (i)}$$

For a point below the surface of the earth

$$g_d = g\left[1 - \frac{x}{R}\right]$$

$$\Rightarrow \quad g_d = g\left[\frac{R - x}{R}\right] \qquad \text{—— (ii)}$$

Dividing equation (i) by equation (ii)

$$\frac{g_h}{g_d} = \left[\frac{R - 2x}{R - x}\right]$$

$$\Rightarrow \quad g_h = g_d\left[\frac{R - 2x}{R - x}\right]$$

Therefore acceleration due to gravity at a point above the surface of the earth is less than the acceleration due to gravity an equidistant point below the surface of the earth.

Graphical representation of the variation

Variation with distance (r) from the centre of the earth (inside the earth)

$$g_{\text{d}} = g\left[1 - \frac{R - r}{R}\right]$$

$$g_{\text{d}} = g\left[\frac{r}{R}\right]$$

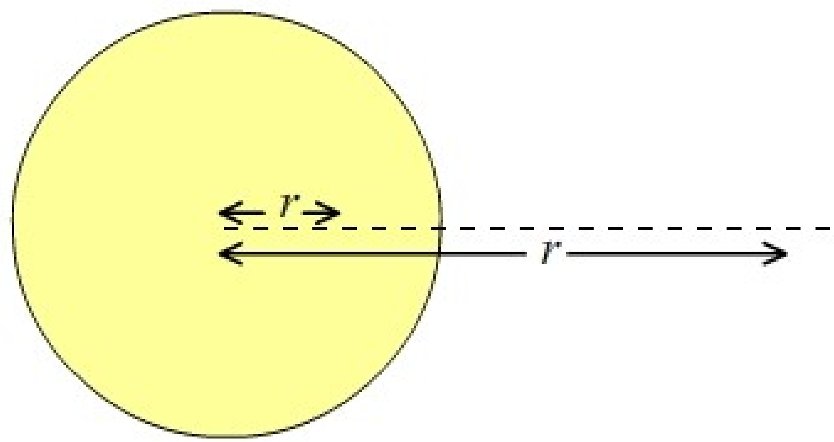
Variation with distance (r) from the centre of the earth (outside the earth)

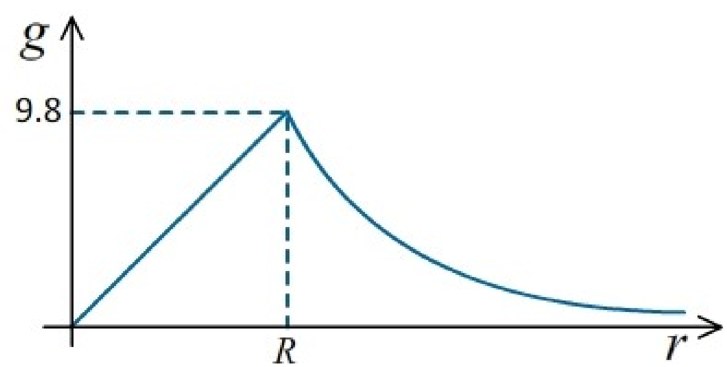
$$g_{\text{h}} = \frac{g}{\left[1 + \frac{h}{R}\right]^2}$$

$$g_{\text{h}} = \frac{g}{\left[\frac{R + h}{R}\right]^2}$$

$$g_{\text{h}} = \frac{gR^2}{(R + h)^2}$$

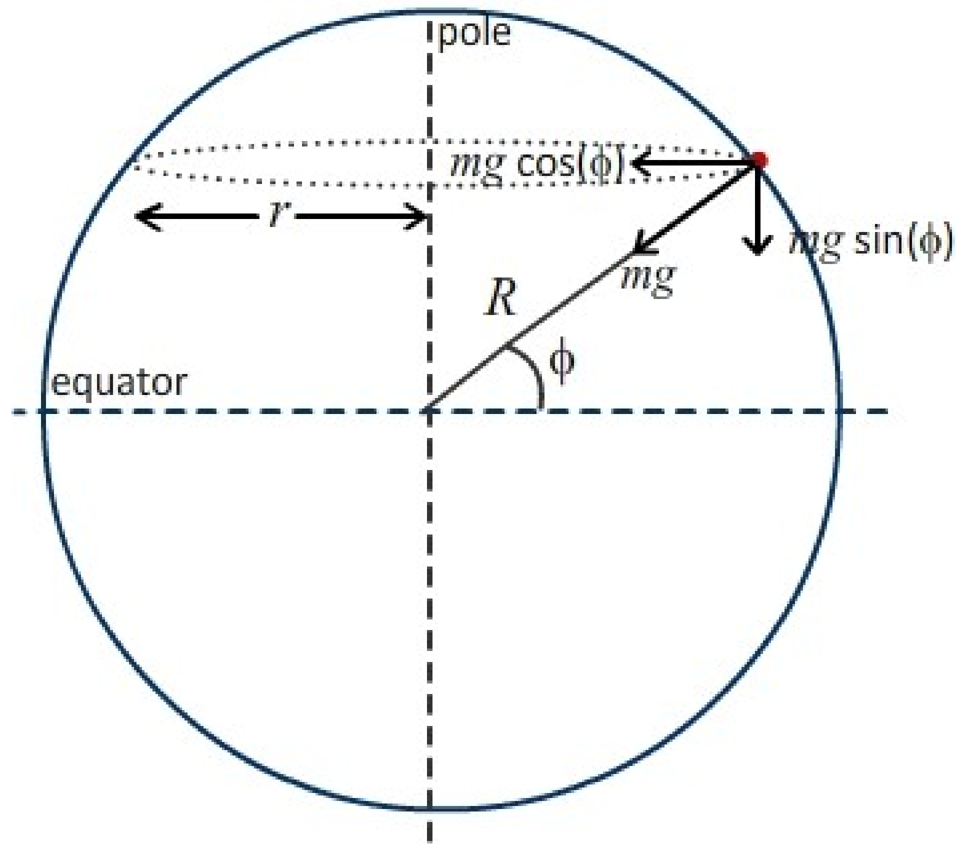
$$g_{\text{h}} = \frac{gR^2}{r^2}$$





Variation of g with latitude (ϕ)

Consider a body of mass m located at a latitude angle of ϕ from the equator. The body would move in a circular orbit of radius r with the same angular velocity (ω) as that of the earth .



From the earth frame of reference, forces acting on the body are
(a) gravitational force
(b) centrifugal force

$$mg_\phi = F_g - F_{CF}(\perp)$$

$$mg_\phi = mg - mr\omega^2\cos(\phi)$$

From figure $r = R\cos(\phi)$ therefore

$$mg_\phi = mg - mR\omega^2\cos^2(\phi)$$

$$g_\phi = g - R\omega^2\cos^2(\phi)$$

g is maximum at poles as $\phi = 90^o$

g is minimum at equator as $\phi = 0^o$

Variation in g as we move from equator to poles

Acceleration due to gravity is effected both due to the shape of the earth and the latitude.
Earth is now a perfect sphere. Its radius is more at the equator and less at poles. This change results in g being less at equators and more at the poles.

As the angle of latitude increases, acceleration due to gravity decreases due the centrifugal force on the body.

$$g_\phi = g - R\omega^2 \cos^2(\phi)$$

Collectively this results in acceleration due to gravity being maximum at the poles and minimum at the equator.

Application of variation of acceleration due to gravity

Using a spring balance :

More sugar can be obtained for the same weight at the equator than at the poles. If the spring balance is calibrated at the equator then it will measure 1kg at the equator.

Sugar is typically sold by mass and balance scales compare mass, for the same weight measured by a spring balance which depends on g. At the poles the acceleration due to gravity is more (due to rotation of the earth and due to non-spherical structure of the earth). Therefore we get less mass of sugar at the poles.

Using a common balance :

Same amount of sugar is obtained at poles or at the equator.

Common balance uses rotational equilibrium. Acceleration due to gravity effects masses in both the pans equally.

Variation due to local conditions

Acceleration due to gravity will be greater at a point above the mountain than in the air at the same height from mean sea level.

This is because at the point on the mountain, the mass of the mountain contributes to the gravitational field locally, increasing the value of g.

At the same height in the air, there is no such additional local mass below the point to exert additional gravitational force. This leads to relatively less gravitational force and results in smaller value of g.

Value of g depends on local conditions such as the concentration of mass near the point under consideration.

Example

As we go from one planet to another, how will (a) the mass and (b) the weight of a body change?

Mass of a body remains constant as it the inherent property of matter.

Weight of the body is due to the net gravitational force acting on it. Forces on the body due to the planets are in opposite directions. Therefore its weight decreases, becomes zero at one point along the line joining the planets and then increases again as it reaches the surface of the other planet.