Oscillations

(displacement, velocity, acceleration and equations of motion)

Note : This PPT will NOT help you learn physics concepts. It is intended only as a quick revision of formulas, definitions, theorems and concepts before examinations. No physics can be learnt just by watching a few videos or going through a few slides of PPT.

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Simple harmonic motion (S H M)

A body is said to be executing SHM if its motion satisfies the following conditions

1. The motion is periodic
2. Motion is to and fro about a mean position
3. Acceleration is proportional to displacement
4. Acceleration and displacement are in opposite directions

Examples

1. Oscillations of a load attached to a spring
2. Oscillations of bob of a simple pendulum
3. Oscillations of a magnetic needle

All periodic motions are not simple harmonic in nature

Examples

1. Revolution of the earth around the sun
2. Revolution of electron around the nucleus

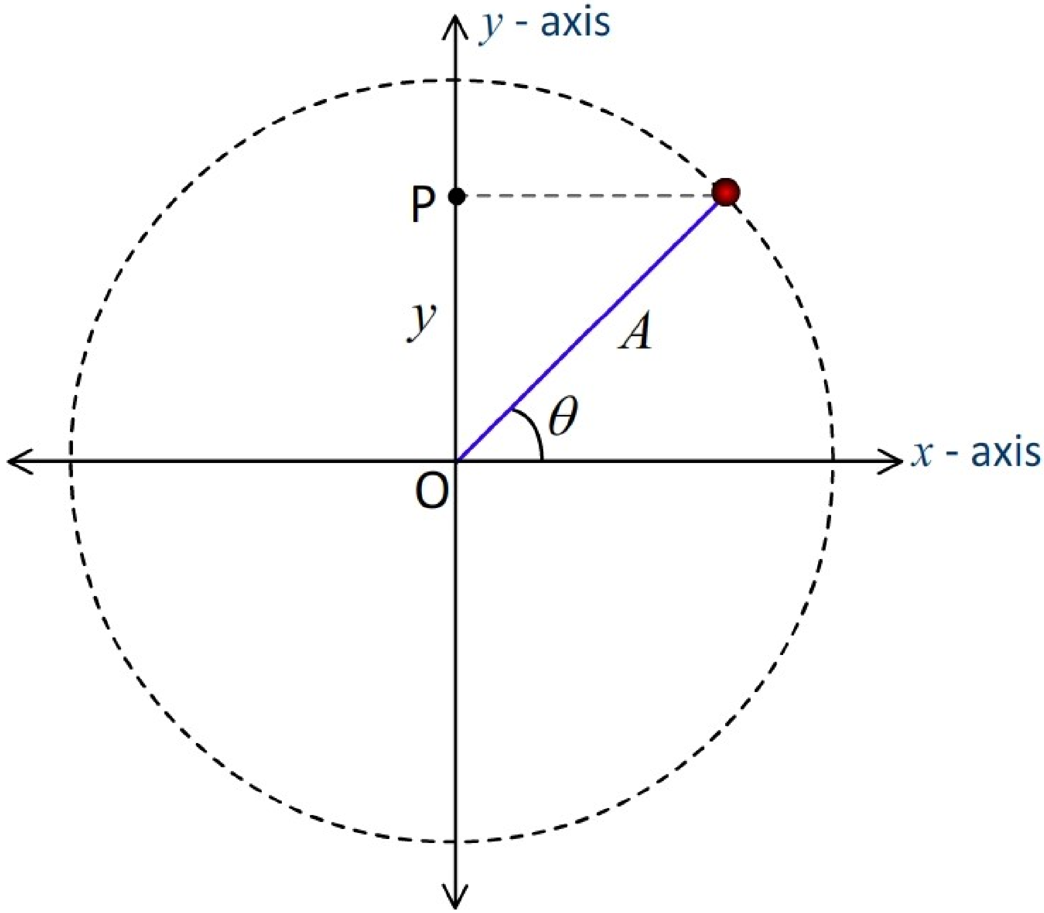
Reference circle (for analyzing SHM)

Consider a body executing uniform circular motion of radius A. Let ω be the uniform angular velocity of the body.

Let P be the foot of perpendicular drawn from the body to the y-axis.

As the body executes circular motion, the point P executes to-fro, periodic oscillatory motion about the mean position (i.e. the centre of the circle).

Let y be the instantaneous displacement of P from the origin at an instant of time (t).



Reference circle simulation

Reference circle (for analyzing SHM)

Displacement (y) of P from O is given by

$$y = A\sin(\theta) \quad \text{(i)}$$

Using the relation $\theta = \omega t$ we get

$$y = A\sin(\omega t)$$

Eq (i) gives displacement of point P as a function of time .

Differentiating eq (i) w.r.t. time we get

$$\frac{dy}{dt} = \frac{dA\sin(\omega t)}{dt}$$

$$v = A\omega\cos(\omega t) \quad \text{(ii)}$$

Eq (ii) gives velocity of point P as a function of time .

Differentiating eq (ii) w.r.t. time we get

$$\frac{dv}{dt} = \frac{d\ A\omega\cos(\omega t)}{dt}$$

$$a = -A\omega^2\sin(\omega t) \quad \text{(iii)}$$

Eq (iii) gives acceleration of point P as a function of time .

Using eq (i) we get

$$a = -\omega^2 y \quad \text{(iv)}$$

From above equation it is observed that

(i) $a \propto x$

(ii) a is in a direction opposite to x

therefore we can say that the point P executes SHM.

Parameters of SHM

$$y = A\sin(\omega t + \phi)$$

- y → Instantaneous displacement
- A → Amplitude
- ω → Angular frequency
- ϕ → Initial phase
- t → time

Amplitude (A) : It is the maximum displacement from mean position

Frequency (n) : It is the number of oscillations completed in a unit interval of time

Time period (T) : It is the time taken for one complete oscillation

Initial phase (ϕ) : It is a parameter that determines the initial state of oscillation

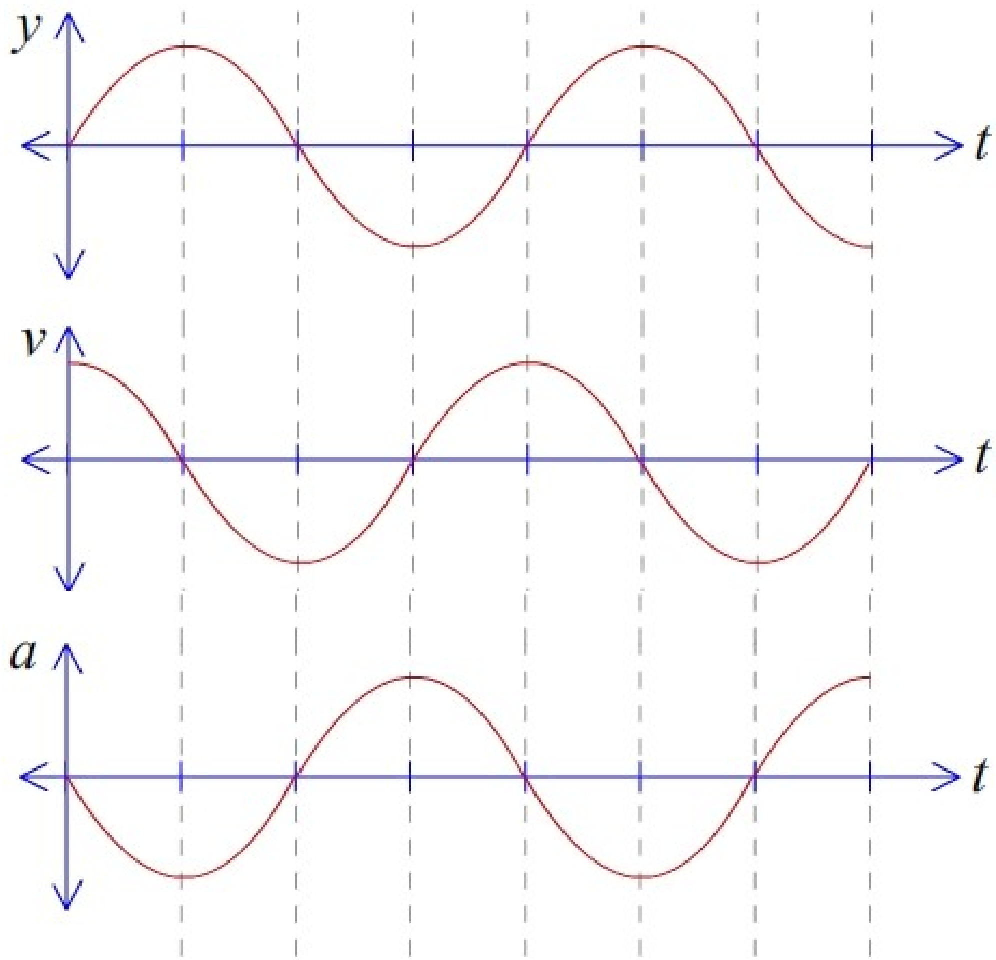
Phase ($\omega t \pm \phi$) : It determines the instantaneous position and velocity of the body

$$\omega = 2\pi n \qquad n = \frac{1}{T}$$

Graphical representations in SHM

Parameters simulation

<u>Phase ($\omega t + \phi$)</u> : It determines the instantaneous position, velocity and acceleration of the body



<u>$y - t$ graph</u>

$$y = A\sin(\omega t)$$

<u>$v - t$ graph</u>

$$v = A\omega\cos(\omega t)$$

$$v = A\omega\sin(\omega t + \pi/2)$$

Phase difference between v & y is $\pi/2$

<u>$a - t$ graph</u>

$$a = -A\omega^2\sin(\omega t)$$

$$a = A\omega^2\sin(\omega t + \pi)$$

Phase difference between a & y is π

Phase difference between a & v is $\pi/2$

Application of SHM equation

The displacement in S.H.M. is given by $y = a \sin(20t + 4)$. What is the displacement when it is increased by$2\pi/\omega$?

Time period of SHM is given by $T = 2\pi/\omega$.
An increase in time by $2\pi/\omega$ implies a time interval equal to time period therefore net displacement of the particle in that time interval is zero.

Velocity as a function of displacement

Velocity as a function of time is given by the relation

$$v = A\omega\cos(\omega t) \quad \text{i}$$

Using the relation

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\Rightarrow \sin^2(\omega t) + \cos^2(\omega t) = 1$$

$$\Rightarrow \cos^2(\omega t) = 1 - \sin^2(\omega t)$$

$$\Rightarrow \cos(\omega t) = \sqrt{1 - \sin^2(\omega t)} \quad \text{ii}$$

Substituting this is eq (i) we get

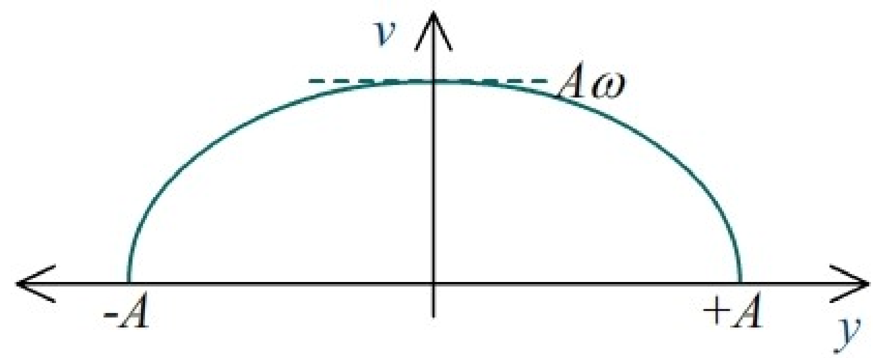
$$v = A\omega\sqrt{1 - \sin^2(\omega t)}$$

$$v = \omega\sqrt{A - A^2\sin^2(\omega t)}$$

Using $y = A\sin(\omega t)$ we get

$$v = \omega\sqrt{A^2 - y^2}$$

Graph of velocity as a function of displacement is a ellipse



Velocity is maximum ($A\omega$) at the mean position (y = 0) and zero at extreme positions ($\pm A$).

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