Oscillations

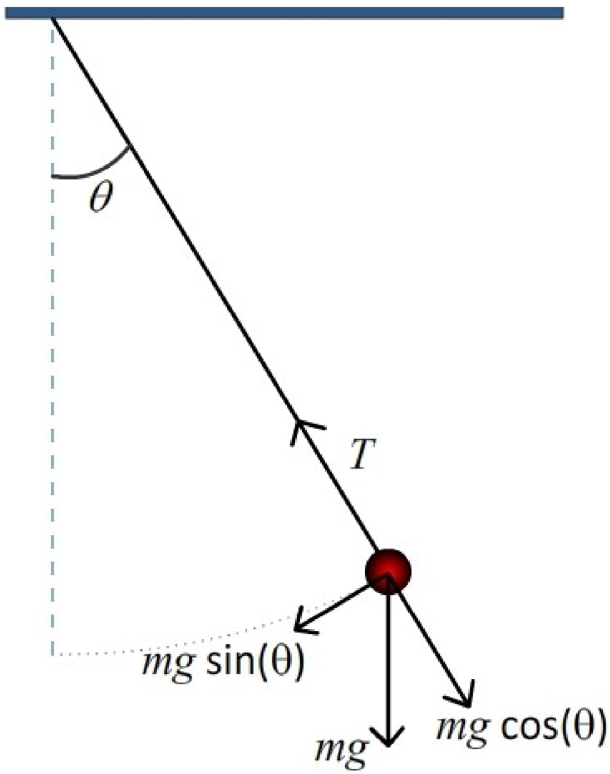
(Simple pendulum, Loaded spring & energy in SHM)

Note : This PPT will NOT help you learn physics concepts. It is intended only as a quick revision of formulas, definitions, theorems and concepts before examinations. No physics can be learnt just by watching a few videos or going through a few slides of PPT.

V Vijaykiran
M.Sc., B.Ed., NET
SIGMA Physics Resource Centre
http://www.sigmaprc.in

Simple pendulum

- A bob (assumed to a point mass) is suspended by a string
- The string is of length l and it is assumed to be inextensible and of negligible mass.
- Effect of other dissipative forces is assumed to be negligible.
- When the bob is displaced from its equilibrium position and released, it executes periodic, oscillatory, to and fro motion about the mean position (O)



- Resultant motion of the bob may be determined by resolving the forces acting on it into components

  (a) parallel to the string

  (b) perpendicular to the string

Simple pendulum simulation

Simple pendulum

Parallel to the string

$$T = mg\cos(\theta)$$

Perpendicular to the string

$$ma = -mg\sin(\theta)$$

$$a = -g\sin(\theta)$$

For small displacements $\sin(\theta) \approx \theta$ therefore

$$a = -g\theta \quad \text{— i}$$

Using relation between linear & angular displacements

$$x = l\theta$$

$$\theta = \frac{x}{l} \quad \text{— ii}$$

$$a = -g\left(\frac{x}{l}\right)$$

$$a = -\left(\frac{g}{l}\right)x \quad \text{— iii}$$

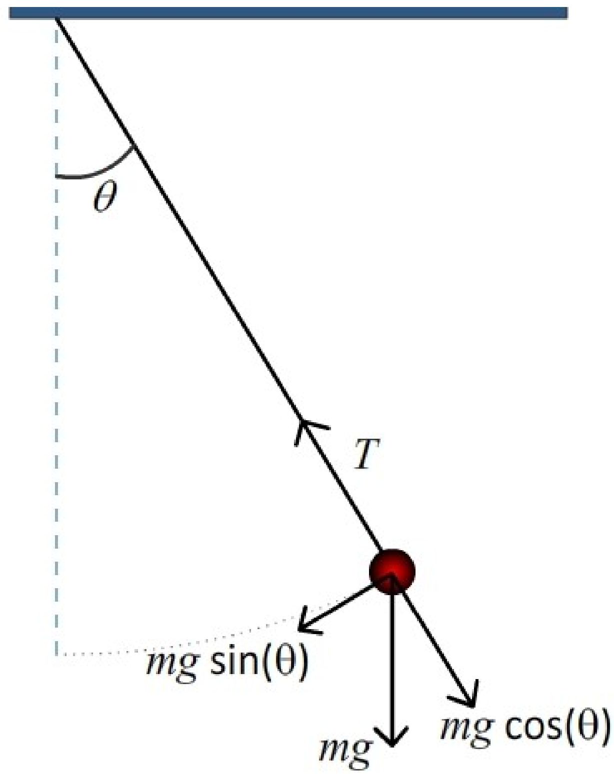
Acceleration in SHM is given by

$$a = -\omega^2 x \quad \text{— iv}$$

Comparing (iii) and (iv) we get

$$\omega^2 = \frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$



$$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Simple pendulum

Seconds pendulum : A pendulum whose time period is 2 seconds is called a seconds pendulum.

Length of a simple pendulum beating seconds is nearly 100 cm or 1m.

Simple pendulum simulation

Application

A girl is swinging seated in a swing. What is the effect on the frequency of oscillation if she stands?

The oscillations of the girl are simple harmonic in nature and similar to that of a simple pendulum. The time period of the oscillation is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

As the girl stands up, the effective length of the pendulum decreases and therefore the time period of oscillations decreases. Therefore the frequency increase.

Application

The bob of a simple pendulum is a hollow sphere filled with water. How will the period of oscillation change, if the water begins to drain out of the hollow sphere?

The time period of the oscillation is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Drainage of water causes a change in the effective length of the pendulum. The time period first increases, reaches a maximum value and then decreases till it becomes returns to its initial value.

As water drains out of the sphere, the centre of mass of the system shifts down leading to an increase in the length of the pendulum. This results in an increase in time period of oscillation.
As the water drains out beyond the half mark, the centre of mass shifts up and eventually returns to the initial position. During this, effective length of the pendulum decreases and finally returns to its initial value.

Application

Will a pendulum clock gain or lose time when taken to the top of a mountain?

Acceleration due to gravity is decreases with increase in height.

The time period of the oscillation is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Therefore the time period of the pendulum decreases

Application

The bob of a simple pendulum is made of wood. What will be the effect on the time period if the wooden bob is replaced by an identical bob of aluminum?

The time period of the oscillation is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Mass of the bob does not effect the value of time period of the pendulum.

Application

What happens to the time period of a simple pendulum if its length is increased up to four times?

The time period of the oscillation is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

As the length is increased to $4l$ we get

$$T' = 2\pi\sqrt{\frac{4l}{g}}$$

$$T' = 2\times 2\pi\sqrt{\frac{l}{g}}$$

$$T' = 2T$$

Application

A pendulum clock gives correct time at the equator. Will it gain or lose time if it is taken to the poles?

A pendulum clock will gain time when taken to the poles.

Time period of the oscillation is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Consider a pendulum clock that shows correct time at equator. Acceleration due to gravity at the poles is more, therefore the time period of oscillations decreases. This implies that the clock runs faster and therefore it is said to *gain time*.

Application

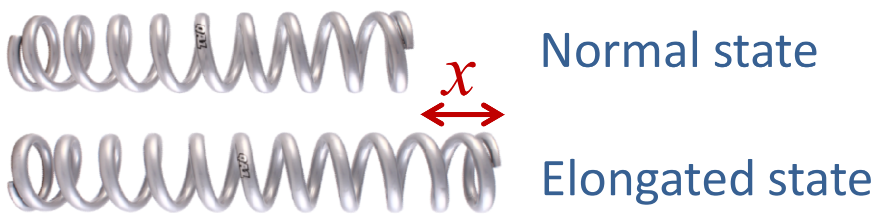
Can a simple pendulum be used in an artificial satellite?

A simple pendulum cannot be used in an artificial satellite as the effective acceleration due to gravity is zero.

Force exerted by a spring

When a spring is either elongated or compressed, it exerts a restoring force that is proportional to the amount of elongation or compression in it.

$$F = -kx$$

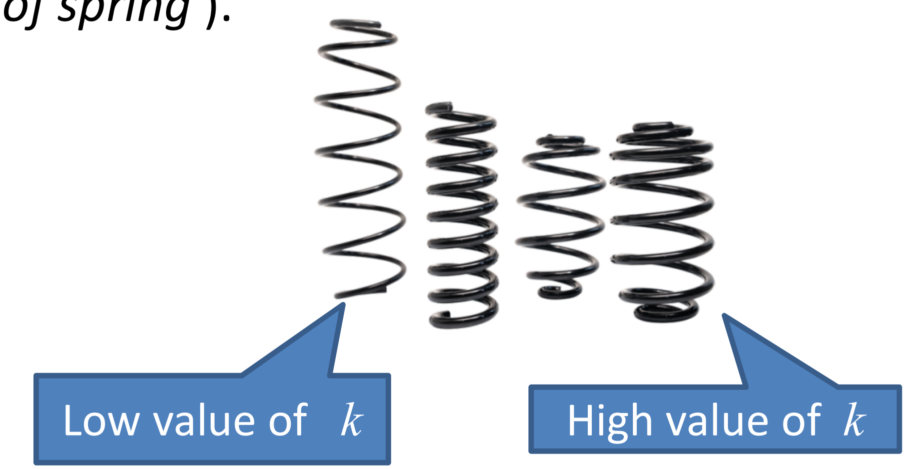


Negative sign indicates that direction of restoring force due to the spring is opposite to extension/compression caused in the spring.

k is spring constant (a measure of *strength of spring*).

Higher value of k implies a *stronger* spring.

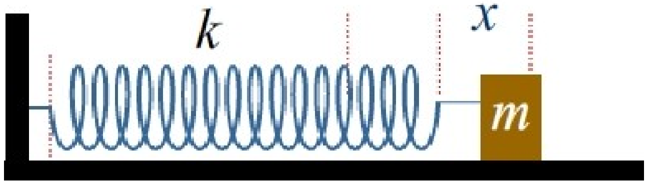
SI unit of k is Nm^{-1}.



Spring pendulum (loaded spring)

Consider a spring of spring constant k placed on a smooth horizontal surface. One end of the spring is fixed to a rigid support and the other end is connected to a body of mass m. The body is displaced causing an extension x in the spring, and the released.

Loaded spring simulation



Restoring force of the spring is $-kx$ therefore

$$F = -kx \quad \text{(i)}$$

From Newton's second law we get

$$F = ma \quad \text{(ii)}$$

Equating (i) and (ii) we get

$$ma = -kx$$

$$a = -\frac{k}{m}x \quad \text{(iii)}$$

The body executes SHM as acceleration is in a direction opposite to displacement and proportional to it.

Acceleration in SHM is given by

$$a = -\omega^2 x \quad \text{(iv)}$$

Comparing equations (iii) and (iv) we get

$$\omega^2 = \frac{k}{m}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Simple pendulum	Spring pendulum
$$T = 2\pi\sqrt{\frac{l}{g}}$$	$$T = 2\pi\sqrt{\frac{m}{k}}$$
Time period is directly proportional to square root of length of pendulum (l)	Time period is directly proportional to square root of mass of the object (m)
Time period is inversely proportional to square root of acceleration due to gravity (g)	Time period is inversely proportional to square root of spring constant (k)
Time period is independent of mass of the object (m)	Time period is independent of acceleration due to gravity (g)
Time period varies with height / depth from the surface of the earth due to change in g.	Time period does not vary with height / depth from the surface of the earth as it is independent of g.

Kinetic energy in SHM

Kinetic energy of a body of mass m having velocity v is given by

$$KE = \frac{1}{2}mv^2 \quad \text{(i)}$$

Velocity of a body executing SHM is given by the relation

$$v = \omega\sqrt{A^2 - y^2} \quad \text{(ii)}$$

Substituting equation (ii) in equation (i) we get

$$KE = \frac{1}{2}m\omega^2\left(A^2 - y^2\right)$$

- KE is maximum at the mean position ($y = 0$)
- $KE_{max} = ½\, m\omega^2 A^2$
- KE is zero that the extreme positions ($y = \pm A$)

Potential energy in SHM

Work done by a force F is causing a displacement dx in a body is given by

$$\mathrm{d}W = F \bullet \mathrm{d}y \quad \text{— i}$$

Force acting on a body executing SHM is given by the relation $F = -k\,y$. Therefore work done by an external force acting on the body is given by

$$\mathrm{d}W = ky\,\mathrm{d}y \quad \text{— ii}$$

Total work done on the body is obtained by integrating the above expression.

$$W = \int_{i}^{f} ky\,\mathrm{d}y$$

$$W = \frac{1}{2} k\,y^2$$

Since the force is conservative, this work done is stored as PE. Therefore

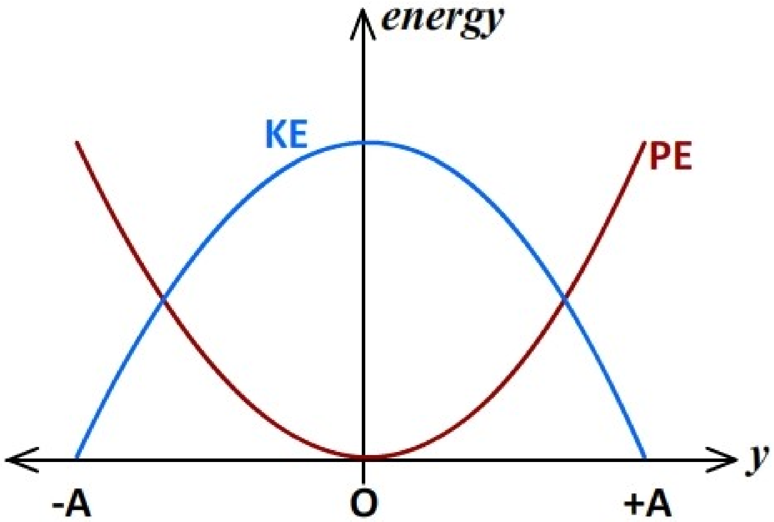
$$PE = \frac{1}{2} k\,y^2$$

Using $k = m\omega^2$ we get

$$PE = \frac{1}{2} m\omega^2 y^2$$

- PE is maximum at the extreme position ($y = \pm A$)
- $PE_{max} = ½\, m\omega^2 A^2$
- PE is zero that the mean positions ($y = 0$)

Energy considerations in SHM



$$PE = \frac{1}{2} m\omega^2 y^2$$

$$KE = \frac{1}{2} m\omega^2 \left(A^2 - y^2\right)$$

$$TE = KE + PE$$

$$TE = \frac{1}{2} m\omega^2 A^2$$

- As the body moves away from mean position, its potential energy increases and its kinetic energy decreases.
- Total energy of the body remains constant at any position
- TE varies with a time period of $T/2$

Energies in SHM simulation

Application

What fraction of the total energy is K.E when the displacement is one half of a amplitude of a particle executing S.H.M?

Total energy of SHM is given by

$$TE = \frac{1}{2} m\omega^2 A^2$$

Kinetic energy of SHM is given by

$$KE = \frac{1}{2} m\omega^2 \left(A^2 - y^2\right)$$

At $y = A/2$ we get

$$KE = \frac{1}{2} m\omega^2 \left(A^2 - \frac{A^2}{4}\right)$$

$$KE = \frac{1}{2} m\omega^2 \left(\frac{3}{4} A^2\right)$$

$$KE = \frac{3}{4} \times \frac{1}{2} m\omega^2 \left(A^2\right)$$

$$KE = \frac{3}{4} TE$$

At $y = A/2$ KE is 3/4th of TE

Application

What happens to the energy of a simple harmonic oscillator if its amplitude is doubled?

Total energy of SHM is given by

$$TE = \frac{1}{2} m\omega^2 A^2$$

When the amplitude is doubled

$$TE = \frac{1}{2} m\omega^2 (2A)^2$$

$$TE = 4 \times \frac{1}{2} m\omega^2 A^2$$

When amplitude is doubled, the total energy increases by 4 times

Comparison of parameters

Parameter	Mean position	Extreme position
Displacement (y)	Zero	$\pm A$ (Maximum)
Velocity (v)	$A\omega$ (Maximum)	zero
Acceleration (a)	Zero	$A\omega^2$ (Maximum)
Kinetic energy	$\frac{1}{2} m\omega^2 A^2$ (Maximum)	zero
Potential energy	zero	$\frac{1}{2} m\omega^2 A^2$ (Maximum)
Total energy	$\frac{1}{2} m\omega^2 A^2$	$\frac{1}{2} m\omega^2 A^2$

sigmaprc@gmail.com

sigmaprc.in