# Oscillations

## ( displacement, velocity, acceleration and equations of motion )

**Note** : This PPT will NOT help you learn physics concepts. It is intended only as a quick revision of formulas, definitions, theorems and concepts before examinations. No physics can be learnt just by watching a few videos or going through a few slides of PPT.

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## Simple harmonic motion ( S H M )

A body is said to be executing SHM if its motion satisfies the following conditions

1. The motion is periodic
2. Motion is to and fro about a mean position
3. Acceleration is proportional to displacement
4. Acceleration and displacement are in opposite directions

Examples

1. Oscillations of a load attached to a spring
2. Oscillations of bob of a simple pendulum
3. Oscillations of a magnetic needle

**All periodic motions are not simple harmonic in nature**

Examples

1. Revolution of the earth around the sun
2. Revolution of electron around the nucleus

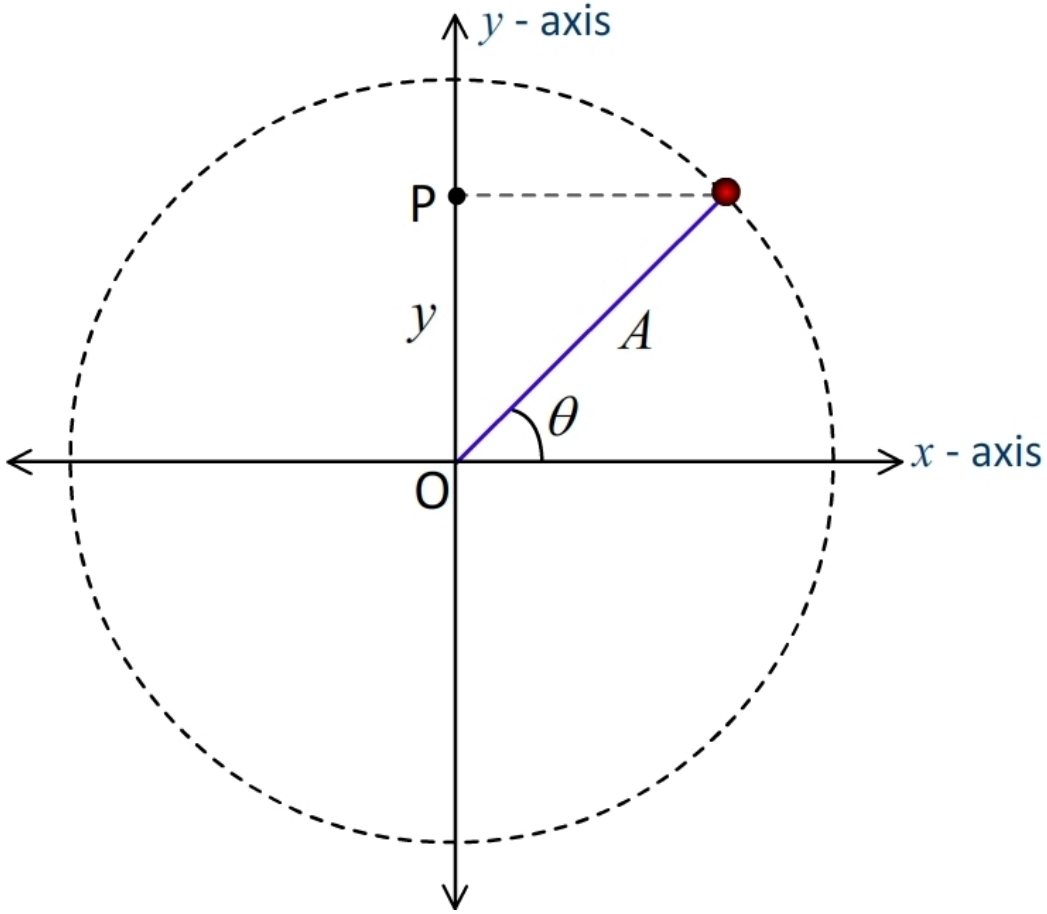
## Reference circle ( for analyzing SHM )

Consider a body executing uniform circular motion of radius $A$. Let $\omega$ be the uniform angular velocity of the body.

Let P be the foot of perpendicular drawn from the body to the $y$-axis.

As the body executes circular motion, the point P executes to-fro, periodic oscillatory motion about the mean position ( i.e. the centre of the circle ).

Let $y$ be the instantaneous displacement of P from the origin at an instant of time ( $t$ ).




Reference circle simulation

**Reference circle ( for analyzing SHM )**

Displacement ( $y$ ) of P from O is given by

$$y = A\sin(\theta) \quad \text{i}$$

Using the relation $\theta = \omega t$ we get

$$y = A\sin(\omega t)$$

Eq ( i ) gives displacement of point P as a function of time .

Differentiating eq (i) w.r.t. time we get

$$\frac{dy}{dt} = \frac{dA\sin(\omega t)}{dt}$$

$$v = A\omega\cos(\omega t) \quad \text{ii}$$

Eq ( ii ) gives velocity of point P as a function of time .

Differentiating eq (ii) w.r.t. time we get

$$\frac{dv}{dt} = \frac{d\ A\omega\cos(\omega t)}{dt}$$

$$a = -A\omega^2\sin(\omega t) \quad \text{iii}$$

Eq ( iii ) gives acceleration of point P as a function of time .

Using eq (i) we get

$$a = -\omega^2 y \quad \text{iv}$$

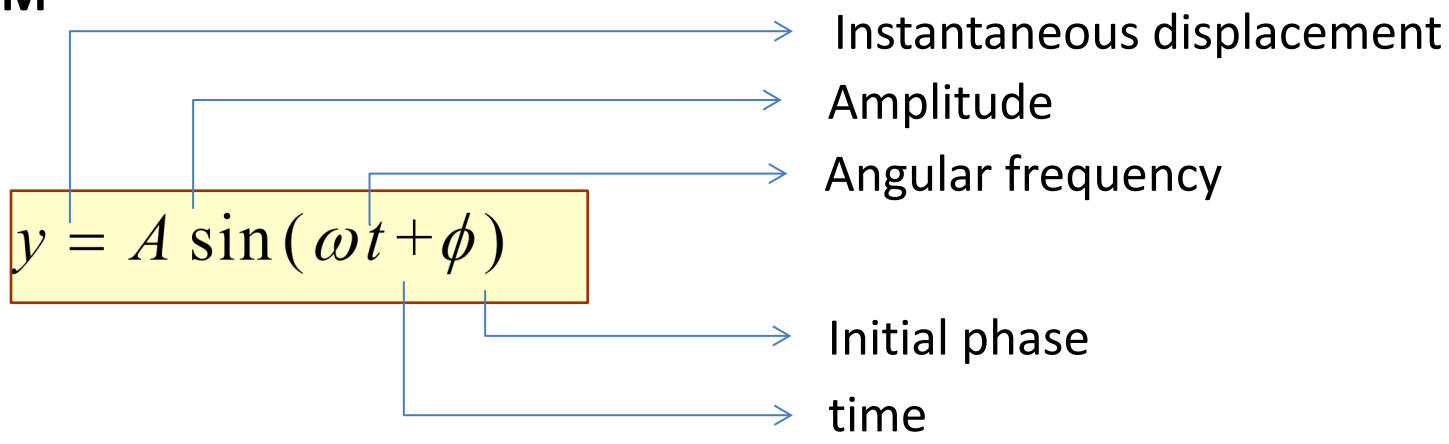
From above equation it is observed that

(i) $a \ \alpha \ x$

(ii) $a$ is in a direction opposite to $x$

therefore we can say that the point P executes SHM.

## Parameters of SHM




**Amplitude ($A$)** : It is the maximum displacement from mean position

**Frequency ($n$)** : It is the number of oscillations completed in a unit interval of time

**Time period ( $T$ )** : It is the time taken for one complete oscillation

**Initial phase** ( $\phi$ ) : It is a parameter that determines the initial state of oscillation

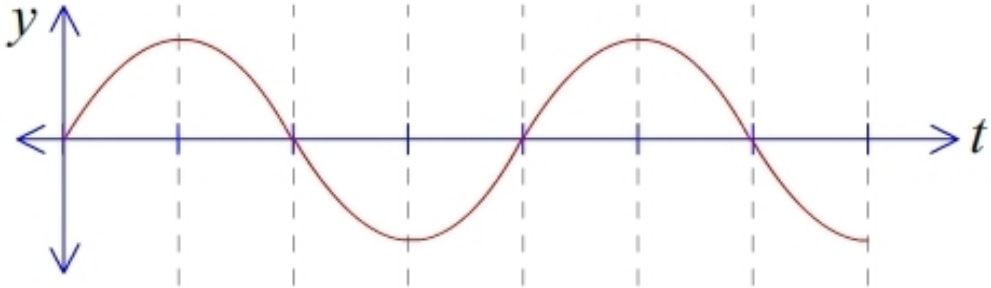
**Phase ( $\omega t \pm \phi$ )** : It determines the instantaneous position and velocity of the body

$$\omega = 2\pi n \qquad n = \frac{1}{T}$$

## Graphical representations in SHM

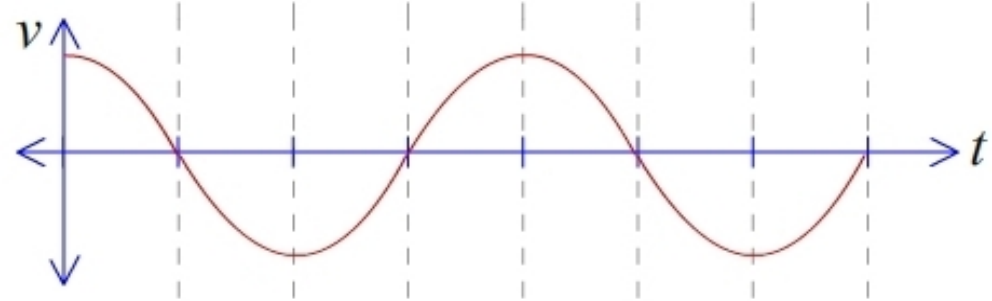
Parameters simulation

Phase ( $\omega t + \phi$ ) : It determines the instantaneous position, velocity and acceleration of the body




**$y - t$ graph**

$$y = A \sin(\omega t)$$

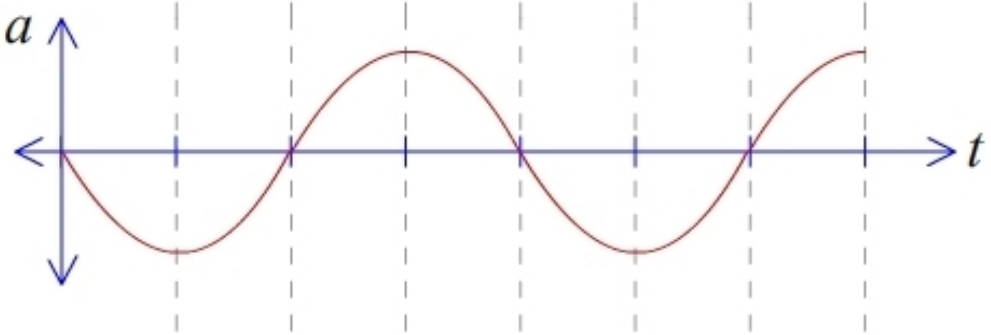



**$v - t$ graph**

$$v = A\omega \cos(\omega t)$$

$$v = A\omega \sin(\omega t + \pi/2)$$

Phase difference between $v$ & $y$ is $\pi$ /2




**$a - t$ graph**

$$a = -A\omega^2 \sin(\omega t)$$

$$a = A\omega^2 \sin(\omega t + \pi)$$

Phase difference between $a$ & $y$ is $\pi$

Phase difference between $a$ & $v$ is $\pi$ /2

**Application of SHM equation**

The displacement in S.H.M. is given by $y = a \sin(20t + 4)$. What is the displacement when it is increased by $2\pi/\omega$?

Time period of SHM is given by $T = 2\pi/\omega$.
An increase in time by $2\pi/\omega$ implies a time interval equal to time period therefore net displacement of the particle in that time interval is zero.

## Velocity as a function of displacement

Velocity as a function of time is given by the relation

$$v = A\omega\cos(\omega t) \quad \text{— i}$$

Using the relation

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\Rightarrow \sin^2(\omega t) + \cos^2(\omega t) = 1$$

$$\Rightarrow \cos^2(\omega t) = 1 - \sin^2(\omega t)$$

$$\Rightarrow \cos(\omega t) = \sqrt{1 - \sin^2(\omega t)} \quad \text{— ii}$$

Substituting this is eq ( i ) we get

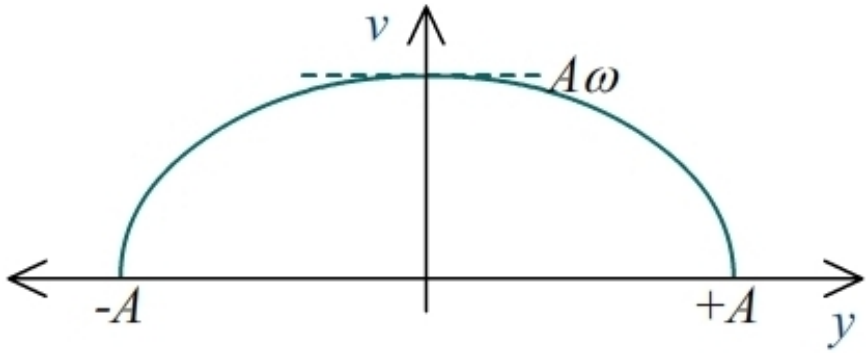
$$v = A\omega\sqrt{1 - \sin^2(\omega t)}$$

$$v = \omega\sqrt{A - A^2\sin^2(\omega t)}$$

Using $y = A\sin(\omega t)$ we get

$$v = \omega\sqrt{A^2 - y^2}$$

Graph of velocity as a function of displacement is a ellipse




Velocity is maximum ( $A\omega$ ) at the mean position ( $y$ = 0 ) and zero at extreme positions ( $\pm A$ ).

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